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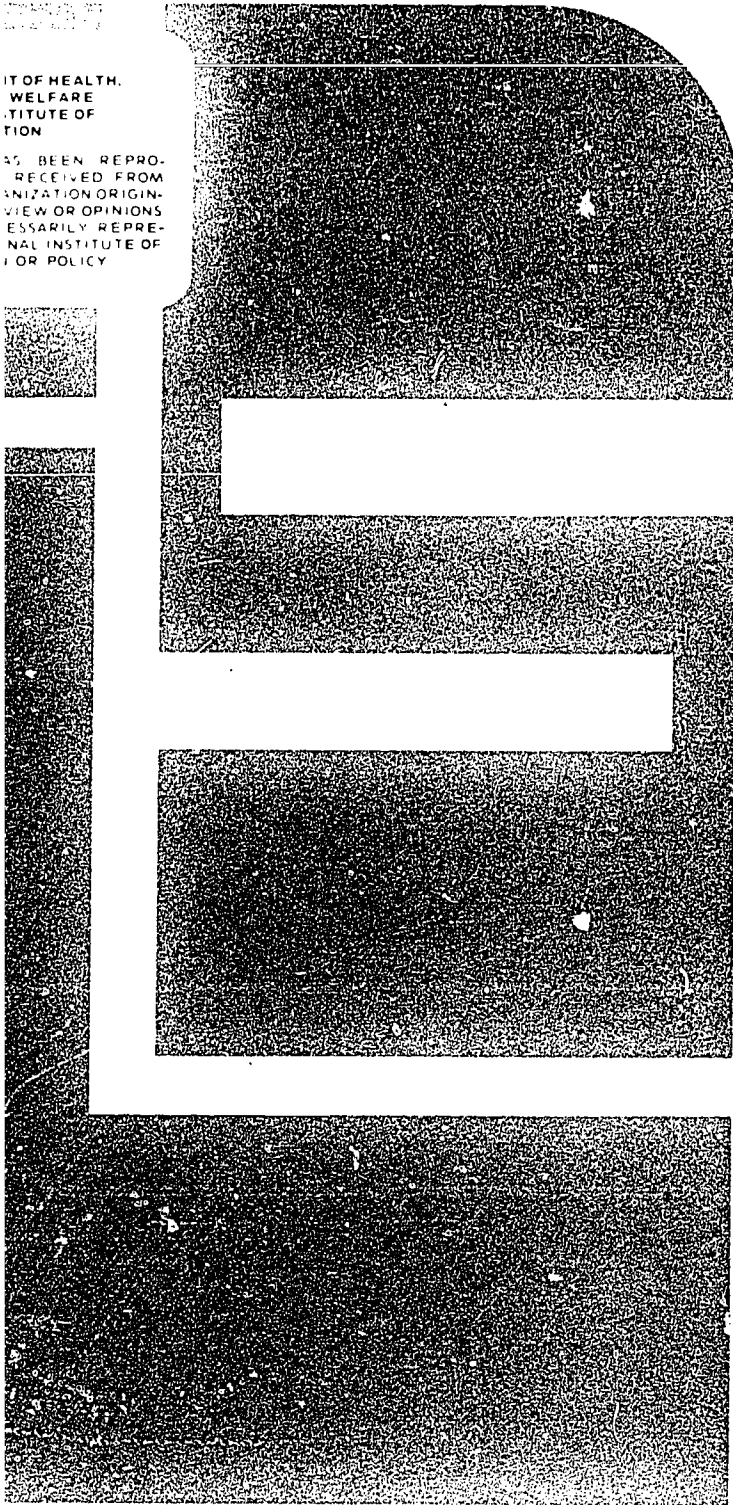
## ABSTRACT

This unit of the Flexible Learning System (FLS), the third of a 3-volume series on children's thinking focuses on the development of quantitative relations in children between 3 and 8 years of age. The series is based on the application of Jean Piaget's work to early childhood education. Quantitative relations concerns all notions of units and number and their application in such areas as measurement and arithmetic. The development of quantitative relations is examined with the use of conservation tasks which reveal children's gradual understanding that the arrangement of objects does not affect their amounts. The unit is designed for use with a group of adults to help them understand quantitative relations and their organization in preconceptual, intuitive and concrete-operational stages of development. Practice in exploring children's thinking is provided. Activities include thought problems, child interviews, discussions, reading, and viewing a color videotape on conservation from a series entitled The Growing Mind: A Piagetian View of Young Children. Rip-out guided interview forms are provided for all interviews. Also included are an introduction to Piaget and his general theory, educational implications of the theory, transcripts of the videotape, and an annotated bibliography. Related FLS units: "Exploring Children's Thinking: Classification"; "Exploring Children's Thinking: Seriation"; "Working with Children's Concepts"; "Using Toys and Games with Children"; "Developing Children's Sense Perception." (Author/SB)

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LEARNER'S GUIDE

**exploring  
children's thinking:  
part 3, conservation**

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ED LEARNING UNITS FOR ADULTS IN EARLY CHILDHOOD EDUCATION

**EXPLORING  
CHILDREN'S THINKING:  
Part 3  
The Development of  
Quantitative Relations  
(Conservation)**

Preschool - Third Grade

by

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3

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## ABOUT THE AUTHORS

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# TABLE OF CONTENTS

<b>OVERVIEW</b> .....	ix
<b>PREFACE</b> .....	xi
<b>INTRODUCTION</b> .....	xv
PIAGET--THE PERSON .....	xv
PIAGET'S VIEW OF KNOWLEDGE .....	xvi
STAGES OF DEVELOPMENT .....	xvii
AN EXAMPLE OF STAGES IN INTELLECTUAL DEVELOPMENT: CONSERVATION .....	xix
THE SOURCES AND DIRECTION OF INTELLECTUAL DEVELOPMENT .....	xx
YOUR AND PIAGET'S EXPLORATION OF CHILDREN'S THINKING .....	xxiv
<b>CHAPTER 1: An Introduction to Quantitative Relations</b> .....	1
WHAT ARE QUANTITATIVE RELATIONS? .....	1
STAGES OF QUANTITATIVE REASONING .....	3
<b>CHAPTER 2: Observing Stages of Reasoning on Conservation Tasks</b> ..	9
A REVIEW OF THE STAGES .....	12
<b>CHAPTER 3: Interviewing Children</b> .....	15
THE FORM OF THE INTERVIEW .....	15

<b>CHAPTER 4: Additional Activities for Exploring the Development of Quantitative Relations .....</b>	<b>29</b>
NUMBER CONSERVATION: PROVOKED CORRESPONDENCE.....	29
ADDITIONAL EXPLORATIONS OF THE CHILD'S UNDERSTANDING OF NUMBER	33
CONSERVATION OF LIQUID AND WEIGHT.....	41
MEASUREMENT OF DISTANCE.....	47
<b>APPENDIX A: Transcript of Videotape .....</b>	<b>51</b>
<b>APPENDIX B: References and Additional Resources .....</b>	<b>61</b>
SOME OF PIAGET'S TRANSLATED WORKS.....	63
PIAGET AS SEEN BY OTHERS.....	65
PIAGET'S THEORY AND EDUCATION.....	67
FILMS AND VIDEOTAPES.....	69



## TABLE OF ACTIVITIES AND FORMS

ACTIVITY 1: Discussion Questions.....	6
ACTIVITY 2: Viewing the Videotape, "The Development of Quantitative Relations".....	9
ACTIVITY 3: After Viewing the Videotape.....	11
ACTIVITY 4: Number Conservation.....	18
ACTIVITY FORM A	21
ACTIVITY 5: Substance Conservation.....	18
ACTIVITY FORM B	23
ACTIVITY 6: Length Conservation.....	18
ACTIVITY FORM C	25
ACTIVITY 7: Discussion Questions After Interviewing Children.....	27
ACTIVITY FORM D: Provoked Number Correspondence.....	31
ACTIVITY FORM E: Part-Whole Relations.....	35
ACTIVITY FORM F: Equalizing Unequal Groups.....	37
ACTIVITY FORM G: Dividing the Whole into Two Equal Groups.....	39
ACTIVITY FORM H: Conservation of Liquid.....	43
ACTIVITY FORM I: Conservation of Weight.....	45
ACTIVITY FORM J: Measurement of Distance.....	49

## OVERVIEW

This unit is about how children between four and eight years of age think about quantities or amounts. It is accompanied by a 30 minute color videotape showing the development of quantitative reasoning. Though this book is designed to be used with a trainer and a group of learners, it will also prove useful to the individual student. There are seven activities designed to acquaint you with quantitative reasoning during the early childhood years. Some of these are discussion activities and some involve interviewing children. The main learning will come from pursuing the activities and sharing your questions and insights with fellow learners. Chapter 4 provides seven additional activities that can be carried out with children to explore their understanding of quantitative relations.

When you have finished this unit, you will:

- be familiar with the general nature of Piaget's work on child development;
- be able to demonstrate the difference between adult- and child-thought, as revealed in a variety of tasks;
- be able to interview children between four and eight years of age to determine their understanding of quantitative relations and their stages of intellectual development;
- be able to demonstrate the difference in the way children handle conservation and measurement problems within the pre-conceptual, intuitive and concrete-operational stages of development;
- be able to recognize how quantitative relations are involved in a wide variety of activities.

## PREFACE

If you plan to affect or are presently affecting the intellectual experiences of children, then you should learn about children's thinking. This book is part of a series called Exploring Children's Thinking (ECT). This series covers four areas of mental development between four and eight years of age: classification, seriation (order relations), number and measurement (quantitative relations), and spatial relations. The first three topics are covered by individual books (Parts 1 through 3) and by corresponding 30 minute color videotapes illustrating children's reasoning. The fourth topic (spatial relations) is covered by a fourth videotape.<sup>1</sup>

In working through this book, you will explore how children develop in their understanding of quantitative relations. Quantitative relations are involved whenever we think about quantities or amounts. All notions of units (inches, miles, quarts, weeks); measurement (time, distance, weight); and numbers (addition, subtraction, counting) involve an understanding of quantitative relations.

If this is your first exposure to this topic, you will be surprised by what you learn. Below a certain stage in development, children do not reason about quantity in the same way as adults, no matter how they are taught or raised. Their ability to reason about quantities, like their thinking in general, changes in fundamental ways as they develop. You will see that children at the same stage of development reason in similar ways,

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1. Videotape series titled: The Growing Mind: A Piagetian View of Young Children.

no matter what their educational or cultural experiences.

The development of quantitative reasoning is revealed in a variety of ways. One of the easiest ways to observe its development is through the use of conservation tasks. Conservation refers to the fact that quantities do not change when they are rearranged or simply changed in appearance. For example, if you pour water from a drinking glass into a bucket, the shape of the water will change, but not its amount. The inability to conserve quantities characterises the early stages of children's thinking about quantity and suggests the problems that young children have with quantitative concepts in general.<sup>2</sup>

Piaget's discovery that children progress through stages in their understanding of conservation and that prior to eight years of age, most children do not conserve, is one of the more widely investigated questions in the field of intellectual development. The general character of Piaget's original findings have been repeatedly supported by the studies of others, carried out with children from a variety of cultures.<sup>3</sup> The more frequently investigated areas concern the conservation of liquid, substance, number, length, weight, and volume.

In the Introduction, we provide a brief description of Piaget's theory and some information about Piaget himself. In addition, it provides an overview of the ECT series and its relationship to Piaget's theory. Much of the Introduction concerns the issue of conservation and its relationship

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2. Piaget, Jean, The Child's Conception of Number.

3. See references to Almy, Dasen, Hyde, Siegel, and Hooper, listed in Appendix B.

to mental development in general and the underlying sources of its understanding. Chapter 1 introduces the topic of quantitative relations and three stages in the development of number conservation--the realization that the number of objects does not change when they are rearranged.

Chapter 2 uses the videotape on quantitative relations to provide an overview of their development between four and eight years of age as revealed in a variety of tasks. In the videotape you will see these tasks administered to children, and you will have an opportunity to discuss the tasks in preparation for your own interviewing activity. Chapter 3 provides some helpful points on interviewing and a number of activity forms for guiding your interviews with children on number, substance, and length conservation. Chapter 4 provides seven additional activity forms for investigating quantitative relations. Three of these describe additional conservation tasks. The appendices contain a transcript of the videotape and a descriptive listing of references and additional readings in the area of child development. Resources on the relationship between mental development and education are also provided.

Now that you have a better sense of what this book is about, we can suggest ways it might help you as a teacher. For one, you will learn to look at children's thinking in a way that reveals its underlying organization. You will come to appreciate the differences between how you and the child view the world. You will learn how to engage children in enjoyable activities that allow you to assess their developmental level. You will come to see similarities and differences between children of the same age, and to find the underlying causes of some of the difficulties children may have with their schoolwork. Ideally, you will be better able

to help children reveal their inner thoughts, and be better able to understand the nature of these thoughts. It is my hope that you find this exploration rewarding.

K.R.A., 1975

# INTRODUCTION

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## PIAGET -- THE PERSON<sup>4</sup>

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This book is one of a series of three dealing with Piaget's study of children's thinking. We selected Piaget's work as our focus because to date he has provided the most complete description and theoretical account of mental development in children.

Jean Piaget was born in Switzerland in 1896 where he has spend most of his life and continues to work. At twenty-two he received his Ph.D. in biology (a field in which he first published at the age of ten) and soon began work in the laboratory of Simon Binet, one of the founders of intelligence testing. While pursuing studies as a biologist, Piaget was developing a dominant interest in knowledge. He began to view its acquisition not as a set of facts and experiences, but rather as an evolutionary process in which knowledge was an outcome of how the mind organizes mental and physical activities. He proposed that the manner in which activities and experiences are organized goes through a series of regular steps or stages.

His early work in Binet's laboratory provided him with much information on the thoughts produced by children. He noticed regular inaccuracies in their thinking that were gradually eliminated with age. On the basis of three papers describing these common inaccuracies, Piaget at the age of twenty-five was made "director of studies" at the Institute J.J. Rousseau in Geneva. He continued his work at the Institute until 1940, at which time he was named Director of the Psychology Laboratory at the University

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4. This Introduction appears in each book in the ECT series.

of Geneva. Along with numerous other posts and duties, Piaget is presently the Director of the International Center for Genetic Epistemology (Geneva), which he founded in 1956.

Throughout this more than fifty years, Piaget has been incredibly productive. He has produced well over two hundred works investigating numerous areas of human knowledge. He has virtually mapped the domain of intelligence from birth to late adolescence and has brought his nearly endless observations into a theoretical perspective drawing from logic, mathematics, physics, biology, psychology, and computer theory.

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#### PIAGET'S VIEW OF KNOWLEDGE

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Piaget's theory has evolved in response to questions asked throughout history: "What is intelligence?" "How are universally true ideas derived?" "Is knowledge really no more than memory?" As the science of psychology developed, it addressed these issues, yielding two views. The first holds that we are born with particular ways of organizing experience, and that knowledge reflects these inborn patterns of organization. The second view is the behavioristic one that has dominated American psychology. It holds that knowledge is a copy of reality and/or learning from others. Piaget has brought a third view to bear, one that strongly suggests the inadequacy of the "inborn" and the behavioristic "copy" theories of knowledge.

As a biologist, Piaget formulated his view around three elements: the organism, the environment, and the interaction between the two.<sup>5</sup> From these

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5. The term "environment" refers to those things that are outside the organism but which affect how the organism functions.



beginning elements come two biological processes that result in change. One is the process of acting on the environment, which is the same as incorporating the environment into actions. Piaget calls this "assimilation" of the environment to the organism. Grasping objects, recognizing a familiar object, and cooking dinner are ways we act upon our environment. The other process is an alteration in the organization of actions as a result of their use. Piaget calls this "accommodation," or the adaptation of actions to the environment. Learning to grasp differently, finding out that something is different than expected, modifying recipes for a meal are examples of how actions are modified through use. "Assimilation" and "accommodation" make up the dynamics of life-- all life being a process of acting on or taking in the environment with resulting changes in the actions themselves and their organization. Changes in the organization or structures underlying actions can be viewed as evolution or development.

Piaget sees knowledge as based in biology. He suggests that the underlying process by which an organism comes to survive is the same as that by which man can arrive at objective knowledge.<sup>6</sup> In both instances the process is composed of the assimilation of reality by the organism and a resulting change in the structures that assimilate. Mathematical thought and primitive biological processes are both based in action systems. The difference is one of the degree of development of those systems.

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#### STAGES OF DEVELOPMENT

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Piaget is probably most widely recognized for his theory that children's

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6. Knowledge that is universally accepted as provable and true.

thinking advances through a series of distinct stages. The essential aspects of a stage theory are that each new stage follows from and depends upon the completion of earlier stages, and that the sequence of development is the same for everyone. Piaget describes a stage in terms of how a child's thinking is organized. The thinking in earlier stages is less well organized than in later stages.

Piaget and his co-workers in Geneva, and a large number of researchers in other countries, have shown that children's thinking in a wide range of knowledge areas goes through a similar developmental pattern. This pattern is described by four major periods. During the first two years, the sensory-motor period, children progress through six distinct stages of intelligence. A second broad period, the pre-operational period, generally lasts between two and eight years of age. During this period, children develop their ability to represent reality with language, imagery, play, drawing, etc., and develop in their understanding of reality. The next period is the concrete-operational stage, during which children develop logical structures (from the adult's view) and apply them to a systematic understanding of a wide range of problems. By early adolescence children enter the formal operational period, considered to be the highest level of mental organization.

STAGE	six stages	Pre-conceptual stage	Intuitive stage	Concrete-operational stage	Formal operational
AGE	0	2	6	8	12
PERIOD	Sensory motor period	Pre-operational period		Concrete-operational period	Formal operational period

It is important to keep in mind that the age at which a child enters or leaves a stage is not specified by the theory. Children of the same age may be in different stages of development. What is so far shown to be true of all children is that all children go through the same series of stages, although not all children progress beyond the concrete-operational stage.

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AN EXAMPLE OF STAGES IN INTELLECTUAL DEVELOPMENT:

CONSERVATION

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One of the more familiar aspects of Piaget's work is the study of conservation. As adults, we recognize that a given amount of something does not change when only its shape has changed. For example, if you pour a tall glass of water into a short fat one, you know that the shape of the water may change, but its amount remains the same. This is called conservation.<sup>7</sup> Conservation is assumed by adults for anything that can be thought of in quantitative terms: a quantity of clay, a measure of distance, a unit of weight, a number of objects, a unit of volume, and so forth.

Piaget and numerous researchers throughout the world have shown that all children progress through the same sequence of stages in their understanding of conservation. Children in the pre-conceptual stage always think that changing the shape or arrangement of objects changes their amount. Children in the next stage believe that quantity is conserved under some circumstances, but not others. By the concrete-operational stage, children firmly believe that changes in shape, arrangement,

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7. Liquid conservation is examined in Chapter 4.

and appearance do not change amount. Furthermore, all children conserve substance (amounts of clay, rice, water, etc.) before they conserve weight (understand that the weight of something does not change when its shape changes); and all children conserve weight before they conserve volume (understand, for example, that a quantity of clay will displace the same amount of water even if the shape of the clay is changed).

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### THE SOURCES AND DIRECTION OF INTELLECTUAL DEVELOPMENT

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Although conservation is only one of many areas of knowledge examined by Piaget, it provides a focus for discussing his theoretical views on how knowledge is acquired.

It has been widely understood that memory, associations between one experience and another, sensory impressions, trial and error learning, and imitations of others, all play a role in thought and affect what we learn. However, each of these, singly or in concert, cannot account for what Piaget and others have found to be true of children's intellectual development. For example, the fact that children think an amount of liquid changes when it is poured, cannot be attributed to a poor memory, to experience, or to the teaching of others. While it is surprising to find children making such judgments, all children think this way at some point in their development.

On the other hand, all children eventually come to know that amounts are conserved, and they do so after passing through the same sequence of earlier stages. When asked why an amount of liquid stays the same when it is poured into a wider container, the conserving child almost universally gives one of the following arguments: "Nothing was added or taken away, so

xx

it's still the same." "The water is now wider than before, but it is also not as tall." "You could pour the water back into the glass and it would be the same as before." These are logically precise arguments for why the amount has not changed even though it looks different. The question naturally arises as to how children come to reason in such systematic terms.

The arguments given by children for why amounts are conserved provide the basis for suggesting some of the likely and unlikely sources of objective knowledge. For example, consider the argument that if nothing is added or subtracted, amounts stay the same. It's easy to imagine that such a principle might be taught, or that it might be experienced through counting activities. However, it is well known that it is virtually impossible to teach this principle to pre-conceptual children; also, children in the intuitive stage either already know or can be taught this with respect to counting, but they do not necessarily apply the principle to other areas of conservation, such as substance and length.<sup>8</sup> Furthermore, all children arrive at an understanding of this principle irrespective of whether it is taught. The intuitive child must be repeatedly convinced of its truth, whereas children a few months older regard it as an obvious fact of nature.

Children who spontaneously understand that amounts do not change when nothing is added or subtracted may just as easily express the argument that liquid poured, for example, from a tall narrow glass into a wide one is conserved, because the water level is now lower, but further around, or wider. This expresses an understanding that changes in one dimension (height) can be compensated by those in another (width). It is unlikely that this principle of compensation was ever taught to most children who express it.

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8. Siegel, Irving and Hooper, Frank (Eds.), Logical Thinking in Children: Research Based on Piaget's Theory.

Without measurement there is no way to tell that changes in one dimension compensate those in another, and the ability to measure, itself, follows rather than precedes an understanding of conservation. When children express an argument of compensation as their basis for conserving, they are simply expressing what they know must be true. As you will see, the argument of compensation is an important clue to their basis for conserving.

A third argument typically given is that if a quantity changes in appearance, there must still be the same amount because it can be changed back to its original appearance. This can be experienced directly. You can pour a glass of water into a bucket and pour from the bucket back into the glass and witness that there is as much water as when you started. But here is an interesting fact. While children may experience this from their first water play on, and while it may even be pointed out to them, this observation does not lead them to conserve. It is not until very close to the concrete-operational stage that such repeated demonstrations lead to an idea of conservation. And, again, children a few months older come to invent this principle for themselves.

The above arguments suggest that an understanding of conservation does not result from experience alone, whether that experience is manipulative and/or social. Piaget has argued that social experiences, physical experiences and maturation (physical growth) are necessary to intellectual development. But they alone are not sufficient to account for something as simple and obvious to adults as conservation.

Piaget has suggested two additional factors that underlie the source and direction of intellectual development.<sup>9</sup> One of these is the coordina-

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9. Ripple, Richard and Rockcastle, Vern N. (Eds.), Piaget Rediscovered: A Report of the Conference on Cognitive Studies and Curriculum Development.

tion of actions and the other is the tendency of this coordination to become reversible.

The concept of a reversible coordination of mental actions is abstract and foreign to most of us. We can give some sense of its meaning by returning to the arguments given by children for conservation. One of the arguments is that a quantity is conserved if nothing is added or taken away. The concept of addition is a mental activity of joining things together. Subtraction is a mental activity of separating. When it is understood that subtracting amounts can exactly compensate adding amounts, then these two mental activities are in a reversible relationship to one another. Such a relationship makes it possible to reason that adding and/or subtracting lead to changes in amounts, and that doing neither leaves amounts the same or conserved. A similar expression of reversible reasoning is demonstrated in the understanding that changes in one dimension can compensate those in another. Changes in height, for example, are reversibly related to changes in width. It is therefore possible for an amount to change in one way and still be the same, because the first change is compensated by a second change. Reversible reasoning is likewise expressed in an understanding that a quantity can be changed in appearance and then changed back to its original form.

Children's understanding of conservation and their supporting arguments do not reflect things that have been taught or recorded from experience. Unlike facts, experiences, or things taught, the principle of conservation cannot be forgotten any more than one can forget that one's brother (sister) also has a brother (sister). Conservation is a product of reversible reasoning applied not just to objects, but to actions upon objects and, more importantly, to internal or mental representations of

actions. An understanding that actions (adding, pouring, narrowing, lengthening, etc.) can be reversed, inevitably results in a conception of conservation which itself is central to all measurement and all conceptions of units. Numbers themselves are simply abstract representations of units that can be counted and separated. And at its core, a unit is no more than a conception of an amount that is conserved in spite of spatial displacements -- changes in arrangements, appearances, and so forth. An ability to conserve is evidence that a child has achieved reversible thought and is capable of thinking of quantity in terms of units that can be measured.

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YOUR AND PIAGET'S EXPLORATION  
OF CHILDREN'S THINKING

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By eight years of age most children begin to evidence dramatic changes in much of their thinking. Piaget and his co-workers have provided a description of these changes in a wide range of areas and have proposed that the emergence of reversible thought is a primary factor in the nature of these changes.

The majority of Piaget's work has concerned "objective knowledge:" knowledge that is subject to proof through agreed-upon arguments. For example, one can prove by the agreed-upon argument of counting, that eight blocks will remain eight blocks, even if their arrangement is changed. There is a range of similar problems that concern areas of quantitative reasoning other than conservation. Some of these are: time, speed-distance-movement, probability, proportionality, geometry, density, force, pressure, and velocity.

In Part 3 of this volume on Exploring Children's Thinking (ECT) we



focus on children's developing understanding of the conservation of number, substance, and length; measurement of distance; and the relation of time, speed and movement. Part 3 consists of a learning unit containing guidelines for exploring children's quantitative thinking and a 30 minute videotape demonstrating the methods of investigation and the developing character of quantitative thought between four and eight years of age.

Other subjects investigated by Piaget are not concerned with quantification. For example, Piaget has studied the understanding of space from early infancy to late adolescence. At some points an understanding of space uses quantitative concepts and at other points it does not. For example, geometry uses units to describe space. A square contains four equal straight lines connected at end points to form an enclosed space. The concepts "four" and "equal" are statements about units and are thus quantitative. However, space can be described without units. For example, the notion of an "enclosed space" does not use any quantitative units.

Part 4 of ECT concerns Piaget's investigation of developing spatial concepts in children between four and eight years of age. Again, we see the same stages as revealed in quantitative thought. Part 4 consists of a videotape demonstrating the methods and results of interviewing children between four and eight years of age on their concepts of straight lines, left-right and foreground-background orientations, and horizontal spatial orientations as demonstrated by the surface of a liquid.

Piaget argues that the similar pattern of stages in quantitative and spatial reasoning results from the general underlying tendency of mental activity to become increasingly organized and reversible. He has attempted to analyze, in terms of reversible classifying and sequencing activities, all that he has demonstrated in spatial and quantitative reasoning, as well

as other areas such as causality and genealogical relations.

In Parts 1 and 2 we explore the development of classifying and sequencing in children between four and eight years of age. Because of the importance attributed by Piaget to these two topics, we have provided a book for each. Part 1 presents a detailed description of the developing understanding of classification and how to explore this development with children. In Part 2 we likewise treat ordinal relations, or the logic of sequences. Each book is accompanied by a 30 minute color videotape.

The topic of classification concerns the coordination of judgments about how objects and events are similar and/or different, and the logic of some and all. For example, all cats and dogs are animals. Because all of the cats are only some of the animals, there are naturally more animals than cats. Piaget argues that the logic of classification is based upon a reversible coordination between combining and separating activities; and that prior to the concrete-operational stage, this reversibility is absent, giving the young child's concepts an illogical appearance. However, it is wrong to call the classification of pre-operational children illogical. It is different from adult thought, yet systematically organized and consistent in its application.

The second topic, ordinal relations, concerns how children coordinate judgments about such things as before-after, first-next, less than-greater than, shorter than-taller than, and so on. Here, as well, there is a logic as expressed in the following: If Steve is older than Leon, and Steve is younger than Pete, then Leon is the youngest. Piaget describes this logic as the reversal of relations such as: If Steve is older than Leon, then Leon is younger than Steve.

Piaget's analysis of knowledge is complex. Some parts are more under-

standable than others and some are more worthwhile to the practicing educator. In this volume on children's thinking we attempt to guide you in a first-hand exploration of part of what Piaget has observed in children's thinking. In pursuing the reading, interviewing, and discussion activities, and by viewing the videotapes, we hope to stimulate not only an appreciation for the character of children's thinking, but a way of looking at thinking itself.

As you work through this volume, you will gradually gain new lenses through which to look not only at children's thinking, but at your own as well. If this topic sparks an interest, you will come to sense the broad patterns of commonality that touch upon a wide range of understandings. In another unit of the Flexible Learning System we help you prepare for exploring children's concepts in general.<sup>10</sup>

Educators commonly ask about the implications of Piaget's theory for education. We address this issue in the concluding chapters of Parts 1 and 2. However, a general view can be expressed quite briefly. Any significant educational implications from Piaget's theory are to ultimately be decided by educators who have come to experience the character of reasoning revealed by Piaget's methods. To translate Piaget's theory to educational prescription must be preceded by an appreciation of what he has discovered. This is the function of the ECT volume.

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10. Alward, Keith R., Working With Children's Concepts, a unit of the FLS.

# CHAPTER 1:

## AN INTRODUCTION TO QUANTITATIVE RELATIONS

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### WHAT ARE QUANTITATIVE RELATIONS?

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Quantitative relations are the relations between amounts or quantities. To say that one thing is less or greater than something else is to state an order relation (See Part 2 of ECT). To say how much one thing is more or less than something else is a matter of quantity. For instance, the radio was invented after the printing press, and the T.V. after the radio. This is a statement of order relations. If we add to this the knowledge that the radio was invented hundreds of years after the printing press, and the T.V., a few decades after the radio, we enter the area of quantitative relations.

The central factor in all quantitative relations is the notion of units of amount. In the above example, we used "years" as our unit to describe amounts of time. You cannot think about quantities without using units. For example, to say that a yardstick is three times as long as a one-foot ruler is to think of the ruler as a unit and the yardstick as three rulers in length. "The man is twice the age of his son;" "This light bulb is 100 times brighter than a candle;" and "Use twice as much rice to double the recipe," are other examples of quantitative statements using units.

Amounts are often described in terms of standard units. For example,

the one-foot ruler is 12 inches long; the yardstick, 36 inches. Light bulbs may be measured in number of watts. Quantities of rice might be described in terms of cups, ounces, sacks, or box-cars. Inches, watts, cups, box-cars, etc., are units of measure.

Quantitative relations are always numerical, because to think about quantities or amounts requires thinking about the number of units involved. Even when we say one thing is equal to another, we use numerical concepts; the first thing is equal to one of the second thing. Even when we say there is no difference between one amount and another, we are using units; the difference between the amounts is no-units or zero units.

The conservation problem mentioned in the Introduction concerns the development of children's understanding of quantitative units. In the remainder of Chapter 3 we will illustrate three stages in the developing understanding of quantitative relations, as seen in most children between four and eight years of age. We will use number conservation as an illustration. In number conservation tasks, the child is typically shown a row of about eight blocks and asked to put out the same number in a similar row. When the child thinks that s/he has put out the same number, the interviewer spreads out the first row and questions the child about the amounts in both rows. "Are there the same number or does one row have more?" As you will see, children at different stages perform very differently on such tasks.

The relationship between conservation and quantitative relations is fairly straightforward. All quantitative notions require an understanding of units. A unit is nothing more than an amount that can be used for determining other amounts. In conservation tasks we see the child's

gradual understanding of amounts that are stable and non-changing; that is, the gradual development of the notion of a quantitative unit. The broader issue of quantitative relations concerns the concept of units and the application of units to the description of quantities.

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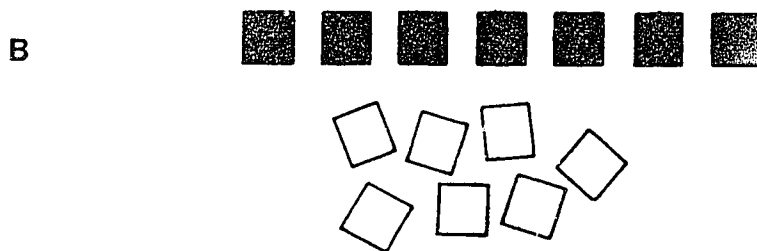
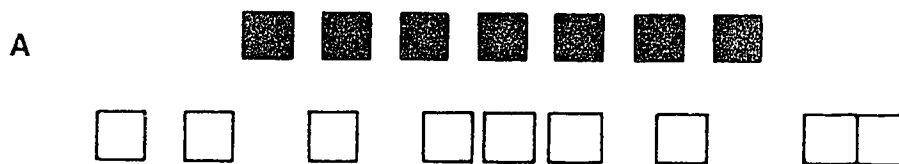
### STAGES OF QUANTITATIVE REASONING

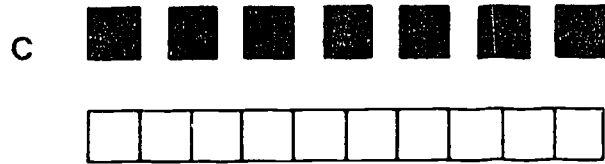
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#### PRE-CONCEPTUAL STAGE (GENERALLY FOUR - FIVE YEARS OF AGE)

Most four to five year olds are still in the pre-conceptual stage of intellectual development. The pre-conceptual stage is a long stage, emerging somewhere around two years of age and generally lasting to about five years of age.

When pre-conceptual children are asked to reproduce the number of objects in a row, they generally do not put out the same number. Instead, they simply make another row of blocks (A), put out a "whole bunch" of blocks (B), or line up the ends of their row with the model (C).





If the child has not put out the same number of blocks, the interviewer may ask the child to count the two rows, and guide the child to correct the number. The interviewer then aligns the two rows in a one-to-one correspondence (so that for every block in one row, there is a block opposite it in the second row), demonstrating, with the child's agreement, that the two rows have the same number. The model row of blocks is then spread apart by the interviewer. The child is then asked if the rows contain the same number of blocks, or if one of the rows contains more blocks. Children that are in the pre-conceptual stage will argue that one row contains more. These children think that if the arrangement of a group of objects changes, their number also changes.

This is very curious reasoning. We know that even if the blocks are spread out, the number must remain the same. However, the pre-conceptual child has no way of considering a relationship between the individual blocks and the whole collection of blocks. When the child thinks about all of the blocks, s/he thinks about their general shape, for instance, how much room they take up. When attention is given to individual blocks, the child forgets about the collection as a whole group. Children during the pre-conceptual period tend to identify quantity with how long a row is. They don't think about number, because that requires thinking about the individual elements and the whole collection at the same time. Pre-conceptual children cannot consider both aspects at the same time, and therefore do not conserve.

### INTUITIVE STAGE (GENERALLY ABOUT SIX YEARS OF AGE)

Most six year olds are in the intuitive stage of intellectual development. During this stage children can determine an equal number of blocks by establishing a one-to-one correspondence (matching blocks one for one). As a result, they correctly copy the number in making their row. However, if one row is spread apart, they think, like children in the previous stage, that the number of objects has changed. Towards the end of this stage, children are not sure whether or not the number of blocks changes when they are rearranged. They fluctuate in their thinking. At one moment they will think that the amounts change, the next moment they think the amounts really stayed the same. Some children will count to find out whether there are the same number. In spite of an ability to count, at this stage children still cannot reason that the number of objects must stay the same if only their arrangement changes and nothing is added or taken away.

### CONCRETE-OPERATIONAL STAGE (SEVEN - EIGHT YEARS OF AGE)

The last stage to concern us is the concrete-operational stage. Children in this stage can reproduce number by constructing a one-to-one correspondence between the model row and their copy. Unlike children at the previous stage, the concrete-operational child also understands that number is necessarily conserved no matter how the objects are rearranged.

The concrete-operational stage is characterized by the child's ability to consider how changes of one nature can exactly counter those of another. For example, addition is countered by subtraction, and in the absence of either, amounts remain the same. Likewise, as a row is stretched out, the



blocks become less tightly bunched or less dense. Furthermore, the changes caused by spreading blocks out can be reversed by bunching the blocks closer together. This ability to think in reversible terms, to think of one action or effect countering another, absent prior to the concrete-operational stage, is what makes it possible to conserve number. As you will see in Chapter 2, this same ability leads to conservation in other areas as well.

This explanation is counter to common sense. It seems much more likely that children learn to conserve because they have been taught or have discovered on their own that amounts do not change when rearranged. However, there is a good deal of evidence to suggest that conservation results not from specific teaching or experience, but from a broad reorganization of the child's thought in general.

This is a good place to review pages xx to xxiv of the Introduction. Additional references on number conservation can be found in Appendix B.

### **Activity 1: Discussion Questions**

In this activity you will have an opportunity to go over some of the important points of Chapter 1.

#### **INSTRUCTIONS**

Think about the following questions and discuss them with others in a small group setting.

1. What are quantitative relations and how are they similar to and different from order relations?

2. How do quantitative relations involve a consideration of units and numbers?
3. Name 15 types of units (such as "inches") used for measuring or describing quantities.
4. Typically, how do four year olds think about number conservation? Five - six year olds? Seven - eight year olds?
5. What are the advances between the pre-conceptual and intuitive stages of number conservation? In what ways is the intuitive stage similar to the concrete-operational stage? In what ways is it different?
6. If you have read Parts 1 or 2 of ECT (or seen the videotape on spatial relations), review the characteristics of these three stages as revealed in classification and/or order relations and/or spatial relations.
7. Try to think of some quantitative problems other than conservation that an intuitive stage or pre-conceptual child would most likely have difficulty solving.
8. Would you try to teach number conservation to a non-conserving child? Why or why not?

## **CHAPTER 2:**

### **OBSERVING STAGES OF REASONING ON CONSERVATION TASKS**

In this chapter, you will view a videotape in which four - eight year old children are interviewed on their understanding of quantitative relations. In the videotape, children are questioned about different kinds of quantity. One task is identical to the task described in Chapter 1 (number conservation). Another task examines the child's conception of quantitative relations applied to a substance such as clay (conservation of substance). Another task involves length (length conservation), and still another investigates the child's ideas about measurement of distance.

#### **Activity 2: Viewing the Videotape, "The Development of Quantitative View of**

#### INSTRUCTIONS

1. Review the following outline of the videotape. Notice the organization of problems and stages.

Child	Age	Stage	Problem
Kevin	4	Pre-conceptual	Number Conservation
Bebe	7	Intuitive	Number Conservation
Shelley	7 3/4	Concrete-Operational	Number Conservation
Clare	4	Pre-conceptual	Clay Conservation
Bebe	7	Intuitive	Clay Conservation
Robin	8	Concrete-Operational	Clay Conservation
Shana	4	Pre-conceptual	Length Conservation
Bebe	7	Intuitive	Length Measurement
Robin	8	Concrete-Operational	Length Measurement

2. Try to observe the ways children think about quantitative relations:

- Concentrate on the different characteristics of reasoning exhibited in different stages.
- Notice how the performances of children at the same stage are similar on the different tasks, and how they are different.
- Think about how the performances of children at different stages are similar and different on the same tasks.

3. Try to determine what might be the best ways to interview children to discover how they think about quantitative relations.

- How do the interviewers pose questions?
- Are some questions better than others? Which ones and why?

You may want to view the videotape twice to focus separately on the children's responses and the interviewing techniques. There is a transcript of the videotape (Appendix A) on which you can take notes as you watch.

### **Activity 3: After Viewing the Videotape**

The questions in this activity are for discussion, and will help you focus on the important points in the interviews.

A. Questions about the children's reasoning:

1. Describe the differences between the (a) pre-conceptual and intuitive stages, and (b) intuitive and concrete-operational stages for each of the four tasks. Justify your answers with observations from the film.
2. Describe the similarities between the (a) pre-conceptual and intuitive stages, and (b) intuitive and concrete-operational stages for each of the four tasks. Justify your answers with observations from the film.
3. What do the children at the pre-conceptual stage have in common in their reasoning on the different problems? At the intuitive stage? At the concrete-operational stage?
4. What do you think a pre-conceptual child would do on the length measurement task? With the crooked and straight paths?

B. Questions about the interviewers:

5. How would you characterize the interviewers' approach to the children's ideas?
6. What were the kinds of questions that led interviewers to discover children's ideas about quantitative relations?
7. What were the kinds of questions that did not lead to understanding more about children's ideas?
8. Overall, what factors lead to a good interview?
9. Overall, what factors lead to an uninformative interview?

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A REVIEW OF THE STAGES

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In general terms, the videotape shows that children's ideas about quantitative relations develop between four and eight years. The development is not a simple result of learning. Only under limited circumstances can children be taught that number, mass, or length are conserved. These concepts are normally developed by children themselves, and this development depends on each child's mental growth in conjunction with his/her own active discovery. Both are important and necessary.

As the videotape shows, children have ideas of their own. While these ideas often seem wrong to adults, they make sense to the children who hold them. In fact, the ideas held by children in each stage are ones that logically and necessarily follow from the general characteristics of the stage.

At the pre-conceptual stage children think about quantity in terms of appearances. The number of blocks is thought of in terms of how spread out they are. The amount of clay is determined by its length. The length of pipe cleaners is determined by which one extends beyond the other. And, in measuring distance, "equal distance" means starting and stopping at similar points without regard for the middle. When we consider the sum of these understandings, we get a picture of the young child's general thinking about quantity. The picture is consistent and makes sense. The pre-conceptual child thinks about quantity as though it had only one dimension. The child does not take into account that blocks have greater distance between them when they are spread apart; that clay is skinnier when it is elongated; that one end of the pipe cleaner is shorter than the other. Children's thinking about quantity at the pre-conceptual stage does not take into account compensatory relations between two dimensions of an amount. Since they think of quantity in terms of isolated aspects of the appearance of things, units of quantity have no meaning for pre-conceptual children.

At the intuitive stage, children begin to take two dimensions into account when they think about quantitative relations. This leads children to fluctuation in their judgments. At this stage, the nature of the number conservation task allows children to experiment by counting in order to find out whether or not quantity is conserved. That is, some children during this stage count before and after the blocks are spread apart in order to find out if the amount stays the same. On the basis of counting they may say that the number has not changed. In other problems like substance (clay) and length, children, of course, can't count the material.

In these tasks, their inability to think of units is clearly revealed.

Because of an incomplete understanding of conservation, children during the intuitive stage do not understand how to use a ruler or other kinds of measuring devices. As with the length of the pipe cleaner, the intuitive child thinks that if the ruler is moved, its length changes. Nor do children during the pre-conceptual and intuitive stages understand the use of units in determining distance. This is revealed in the measurement of distance task. Rather than measuring the distance the doll walks, the child "measures" whether or not the dolls are evenly aligned, and ignores the fact that the two roads are different in length.

At the concrete-operational stage children consider two dimensions when they reason about quantitative relations. This new form of reasoning leads them to the conclusion that quantity remains the same if nothing is added or subtracted. For example, in the number conservation problems, concrete-operational children know that there must be the same number, and they know this without counting. They reason that even though the line of spread elements is longer, there is more space between elements and the number remains the same. In the clay problem, children know that even though the clay is elongated, it's also thinner, leaving the amount the same. Similarly, in the length problem, children know that even though one pipe cleaner extends beyond the other at one end, that difference is compensated on the other end. This understanding of conservation forms a basis for children using units of measure when they consider distance.



## **CHAPTER 3:**

### **INTERVIEWING CHILDREN**

By this time you should have a fairly good idea of how children reason about quantitative relations. This chapter will prepare you to interview children on conservation tasks. You will be given a general guide on how to conduct an interview. Then you will be provided procedures for each of the three conservation tasks shown in the videotape. You'll first practice administering these tasks to a classmate. Then you'll interview children. Following the interview experience you will return to class for a group discussion of your interviewing experiences.

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#### THE FORM OF THE INTERVIEW

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A good interview is not the result of following a prescribed procedure word for word. It does not result from following a set of specific instructions. Rather, a good interview is created by (1) being attentive to what the child is doing; (2) being attentive to how the child is understanding your instructions; and (3) forming "educated guesses" about how the child is reasoning. From these "educated guesses" you should be able to generate additional questions in order to determine how the child is actually reasoning. Your development as an interviewer depends upon all of these things. This skill takes time and practice.

Here is some general advice about interviewing which may be helpful.

## Do's and Don'ts of Interviewing

1. It is important that you enter the interview with the right attitude. This should be a curiosity about children's ideas about quantitative relations and a willingness to explore ways of finding out. Do not be concerned with whether or not children have the "right" answers. If given the opportunity, children will do as well as they can. The object is to find out how children reason.

A typical mistake of beginning interviewers is to try to get the best, most advanced response from the child. This is understandable, but it inevitably results in hounding the child, making instructions too complex, and generally communicating to the child that you want something s/he is not giving you, that there is something wrong with his/her thinking. It's better to go slow and let the child take the lead with as little interference from you as possible. The child's reasoning is a sensitive and personal issue and you should maintain a respect for it throughout the interview.

2. Present the conservation materials to the child in a natural manner, saying something like, "Let's work with these for awhile," or "I have some things I want you to do with these." The tasks will generally strike children as natural and reasonable. Rather than providing a lot of explanations about what it's all about, just get going.

3. Remain as flexible as possible throughout the interview. If the child does not immediately respond to your questions, wait and ask again. If the child does not do what you ask, let him/her fool around a bit. The child may need some time to feel comfortable with the materials. If the child comments on things, even if they do not relate to the task, listen and respond to what is said. When you feel that the child is drifting from the focus, remind him/her of what you want. The experience itself should be pleasurable for you and the child. If you feel uncomfortable, or the child is feeling anxious and uncomfortable, then it is best to discontinue the interview. If children give any indication of wanting to quit, honor their choice.
4. Try not to be anxious with yourself or the child. You can and will make "mistakes," fail to ask the "right" questions, misunderstand the child and so on. It takes experience to become a good interviewer. You will have to make mistakes to learn. So don't worry about it. If you worry, you'll probably worry the child as well. Children like to share their thoughts with adults, if adults seem interested and respectful.
5. Remember, you are the adult, it's your interview and you're in control. Be clear to the child about what you want. If you let the child "take over," s/he will do whatever strikes his/her fancy and you will have learned little. For example, if the child starts arranging the blocks in a design, say, "You can play with the blocks in any way you like after we've finished, but now

I want you to...." Or, mold the child's spontaneous activity to your interview concerns. For example, question the child about conservation after s/he has rearranged the material.

Once you have gained a working familiarity with these ideas for how best to interview children, you can put them into practice by completing the next three activities. You will interview first an adult partner, then a child on each task. The best way to conduct interviews, at least in the beginning, is in pairs. One person interviews; the other person sits in the background to record what is being said and what goes on. This frees the interviewer to fully attend to the child. Use this approach when practicing with adults and when interviewing children. If a tape recorder is available, it can be used instead of a partner. However, the interviewer should take notes on the manipulations of the materials, responses, etc. Later the tape recording and the notes should be integrated into a "protocol," a convenient, concise record of the important features of the interview.

Each activity is represented by an activity form:

**Activity 4: Number Conservation** - Activity Form A

**Activity 5: Substance Conservation** - Activity Form B

**Activity 6: Length Conservation** - Activity Form C

On these forms you will find a list of the materials needed for each task, the purpose of the task, and the method of presentation. Feel free to use any similar materials that may be more convenient for you. You will also find outlines for suggested interviews. Remember that these are only general guides. Children will often give responses for which the outline does not provide follow-up questions. In order to determine how the child

is reasoning, you will have to think of many of your own questions with which to further probe and explore. These follow-up questions are essential for getting a real understanding of the way the child is thinking. Copies of these activity forms can be used in the future as a means of keeping records on children's performance. Or you may wish to devise your own. The important thing is to record all the information that is necessary for understanding how the performance of the task reflects the way the child reasons.

ACTIVITY FORM A: Number Conservation

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

MATERIALS: At least 20 one inch cube-blocks, half of one color and half of another (other objects can be used instead of blocks).

PURPOSE: To see how the child determines an equal number of blocks, and to see if the child thinks number is conserved when the blocks are rearranged.

PRESENTATION: Put eight blocks in a straight row. Put the remaining ones off to the side, available to the child.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

WHICH BLOCKS DO YOU WANT? O.K.,  
THESE WILL BE MINE. I'M GOING  
TO PUT SOME OF MINE IN A ROW. PUT  
OUT THE SAME NUMBER OF YOUR  
BLOCKS, SO WE BOTH HAVE JUST AS  
MANY.

ARE YOU SURE THAT'S JUST AS MUCH?  
HOW DO YOU KNOW?

If the child has not constructed a one-to-one correspondence, arrange the blocks so that each block in one row is opposite each block in the other row.

NOW, DO WE BOTH HAVE THE SAME NUMBER?

Spread one row apart as the following is stated

WATCH WHAT I AM DOING. NOW, DO WE BOTH HAVE THE SAME NUMBER, OR DOES ONE OF US HAVE MORE? HOW DO YOU KNOW?

If the child conserves (says there is still the same number), give a countersuggestion such as, ANOTHER BOY (GIRL) SAID THAT THIS ONE WAS MORE BECAUSE IT'S LONGER. DO YOU THINK HE (SHE) WAS RIGHT OR WRONG? HOW DO YOU KNOW?

OR

If the child does not conserve (says one now has more), give a counter suggestion such as, ANOTHER BOY (GIRL) SAID THERE WERE STILL THE SAME NUMBER BECAUSE WE DIDN'T ADD OR TAKE ANY BLOCKS AWAY. DO YOU THINK HE (SHE) WAS RIGHT? HOW DO YOU KNOW?

47

ACTIVITY FORM B: Substance (Clay) Conservation

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

MATERIALS: Two cups of playdough, each of a different color.

PURPOSE: To see whether the child understands that changing the shape of a substance does not alter its amount.

PRESENTATION: Present two balls of playdough, each containing about 3/4 cup.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

DO THESE BALLS OF PLAYDOUGH HAVE JUST AS MUCH OR DOES ONE HAVE MORE? HOW DO YOU KNOW? (If child claims they are unequal, help him/her make them the same).

ARE YOU SURE THEY HAVE THE SAME AMOUNTS NOW? HOW DO YOU KNOW?

I'M GOING TO ROLL ONE OF THESE OUT TO MAKE A LONG, SKINNY SNAKE. DO BOTH PIECES STILL HAVE JUST AS MUCH CLAY OR DOES ONE HAVE MORE THAN THE OTHER?

48

23



EXPLAIN WHY THAT'S SO.

If the child conserves (says there's still the same amount), give a counter suggestion such as: A LITTLE BOY (GIRL) TOLD ME THAT BECAUSE THIS ONE IS SO LONG THERE'S REALLY MORE CLAY HERE. DO YOU THINK HE (SHE) WAS RIGHT? WHY?

OR

If the child does not conserve (says there is now a different amount), give a counter suggestion such as: A LITTLE BOY (GIRL) TOLD ME THAT THEY'RE STILL THE SAME BECAUSE WE DIDN'T ADD OR TAKE ANY CLAY AWAY. DO YOU THINK HE (SHE) WAS RIGHT? WHY?

NOW I'M GOING TO ROLL THE SNAKE BACK INTO A BALL AGAIN. ARE THEY BOTH THE SAME NOW? DO THEY BOTH HAVE THE SAME AMOUNT OF PLAYDOUGH? HOW CAN YOU TELL?

(You may wish to change the shape of one ball in other ways, e.g., flattening it out into a disc, a doughnut shape, etc.)

ACTIVITY FORM C: Length Conservation

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

**MATERIALS:** Five pipe cleaners: two six-inch, three others that are shorter and longer than six inches (sticks, wire, etc. may be used instead of pipe cleaners).

**PURPOSE:** To see if the child can select materials that are the same length, and to see if the length judgment changes as the spacing or shape changes.

**PRESENTATION:** Put out all the pipe cleaners and hold up a six-inch one. Ask the child to select another that is the same length - just as long. Give him/her the standard to use. Let the child handle the pipe cleaners.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

(When the child selects a wire)

HOW DO YOU KNOW THEY'RE THE SAME LENGTH?

Put the two pipe cleaners down next to each other so that they are even.

IF I MOVE ONE THIS WAY (so that the ends do not match), ARE THEY STILL THE SAME LENGTH OR IS ONE LONGER? WHY?

If the child conserves (says they're still the same length), give a counter suggestion such as:  
A LITTLE BOY (GIRL) TOLD ME THAT BECAUSE THIS ONE STICKS OUT HERE (pointing to an extending end) THAT THIS ONE IS LONGER. IS THAT RIGHT? WHY?

OR

If the child does not conserve (says they're different lengths), give a counter suggestion such as:  
A LITTLE BOY (GIRL) TOLD ME THAT BECAUSE WE ONLY MOVED THEM, THEY'RE STILL THE SAME LENGTH. IS THAT RIGHT? WHY?

Match the ends again.

ARE THEY THE SAME NOW?

If the child conserves length, try the following task:  
I'M GOING TO TAKE MY WIRE AND MAKE A CROOKED ROAD, LIKE THIS."

ARE THEY STILL THE SAME LENGTH OR IS ONE LONGER? WHY?



IF YOU WERE TO WALK DOWN THIS CROOKED ROAD AND I WALKED DOWN THE STRAIGHT ROAD, WOULD WE BOTH HAVE THE SAME DISTANCE TO WALK OR WOULD ONE OF US WALK FURTHER? WHY?

## Activity 7: Discussion Questions After Interviewing Children

Each individual should present the results of his/her interviewing for group discussion. Try to address the following questions:

1. What went well?
2. What went poorly?
3. What stages did you find?
4. On what basis did you identify children as being at a particular stage?
5. How could your interview have been improved?
6. What follow-up questions did you ask that were not on Forms A-C?
7. Should you have asked additional follow-up questions? Why?

## CHAPTER 4:

### ADDITIONAL ACTIVITIES FOR EXPLORING THE DEVELOPMENT OF QUANTITATIVE RELATIONS

Chapter 4 provides a variety of additional activities that can be used for exploring children's understanding of quantitative relations. Three of these are additional conservation tasks. The remainder reveal other aspects of quantitative reasoning, including the length measurement problem seen in the videotape.

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#### NUMBER CONSERVATION: PROVOKED CORRESPONDENCE

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Many of the natural objects in the child's environment are such that one thing belongs with another; cups and saucers, candles in candle holders, flowers in vases, eggs in egg cups, and so forth. It's possible to see if such familiar occurrences make it easier for a child to establish equal sets of objects and to appreciate the conservation of their numbers. Piaget's general findings are that the use of such objects makes it possible for children to conserve number somewhat earlier but it does not effect conservation of number with other materials nor the child's general development of quantitative relations. You should try this task with your younger children to see if there is a difference. See The Child's Conception of Number<sup>11</sup> for more information.

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11. Piaget, Jean, The Child's Conception of Number, pages 41-56.

ACTIVITY FORM D: Provoked Number Correspondence

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

DO NOT DO THIS TASK WITH CHILDREN WHO CONSERVE NUMBER IN ACTIVITY 4

MATERIALS: 16 toy saucers and 16 toy cups.

PURPOSE: To see how the child establishes the same number of cups and saucers; to see what the effect of the spatial arrangement is on establishing an equivalent number and maintaining it after rearrangements.

PRESENTATION: Set out eight saucers fairly close together in a straight line.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

WE'RE GOING TO HAVE A PARTY,  
YOU NEED TO GET ENOUGH CUPS  
TO GO WITH THESE SAUCERS.

(Try to avoid suggesting  
putting the cups on the saucers.)

DO ALL THE CUPS HAVE SAUCERS?  
HOW CAN YOU SHOW ME? MAKE THEM  
SO THAT I CAN SEE THERE'S A CUP  
FOR EVERY SAUCER.

If the child does not feel they are the same or if they are not the same, have him/her correct the situation. If s/he cannot establish the one-to-one correspondence by putting a saucer with each cup, help him/her.

NOW ARE THERE THE SAME NUMBER OF CUPS AS SAUCERS OR IS ONE MORE?

NOW I TAKE OFF ALL THE CUPS AND PUT THEM TOGETHER. Put the cups closer together in a straight row. IS THERE STILL A CUP FOR EVERY SAUCER?

If the child does not conserve, ask what has to be done to make them the same.

LET'S PUT THEM BACK ON THE SAUCERS TO SEE IF THERE ARE THE SAME NUMBER. ARE THERE THE SAME NUMBER OF CUPS AND SAUCERS OR ARE THERE MORE CUPS OR MORE SAUCERS?

Rearrange again by stacking all the saucers in a pile. ARE THERE MORE CUPS OR SAUCERS OR IS THERE THE SAME NUMBER OF CUPS AND SAUCERS? WHY OR WHY NOT? WHAT MUST WE DO TO HAVE THE SAME AMOUNT?

If the child gives a non-conserving response, at some point ask: ARE THERE REALLY MORE CUPS OR DOES IT JUST LOOK LIKE MORE?

It's possible to set up a situation where the length of the two lines correspond but not the number of elements and the child is asked if there are just as many cups as saucers.

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## ADDITIONAL EXPLORATIONS OF THE CHILD'S UNDERSTANDING OF NUMBER

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In the following, we present three additional tasks that can be used for exploring children's thinking about number. The first problem (Part-Whole Relations) taps the child's understanding that a quantity remains the same even though it is divided into parts. For more information on this task, refer to pages 185-190 of The Child's Conception of Number.

In the second task (Equalizing Unequal Groups), we examine the child's understanding that as objects are moved from one group to another, the number of objects in the first group decreases while that of the second group increases. For more information, see pages 190-195 of The Child's Conception of Number.

The third task (Dividing a Whole into Two Equal Groups) explores how children go about dividing a number of objects into two equal groups. Pages 195-198 of The Child's Conception of Number provide more information on this task.



ACTIVITY FORM E: Part-Whole Relations

Name \_\_\_\_\_ Child's Name \_\_\_\_\_

Date \_\_\_\_\_ Child's Age \_\_\_\_\_

Child's Teacher \_\_\_\_\_

MATERIAL: Eight counters of one color and eight of another (e.g., eight black and eight white).

PURPOSE: To determine whether the child can see that the addition of two subgroups is equal to their sum; or that a quantity divided into parts still exists as an addition of the parts.

PRESENTATION: Put out the eight white counters in two symmetrical groups of four each. Put out the black counters in a similar arrangement.



OUTLINE OF INTERVIEW

CHILD'S RESPONSES

THESE ARE YOUR TREATS TO EAT ON ONE DAY (white) AND THESE (black) ARE YOUR TREATS TO EAT ON THE NEXT DAY. DO YOU HAVE THE SAME NUMBER OF TREATS TO EAT ON THIS DAY AS ON THE OTHER DAY? WHY OR WHY NOT?

If the child says they are not the same: MAKE THEM THE SAME SO YOU HAVE JUST AS MANY TREATS HERE (white) AS HERE (black).

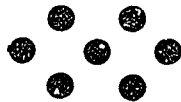
ON THE FIRST DAY (white) YOU DECIDE TO HAVE FOUR TREATS IN THE MORNING AND THE OTHERS AT NIGHT. ON THE NEXT DAY (black) YOU ARE FULL AND DECIDE TO HAVE ONE IN THE MORNING AND THE REST AT NIGHT. (Take three of the black counters from one group and put them with the four other black counters making  $1+7=8$ .)

○ ○



○ ○

○ ○



○ ○

DO YOU HAVE AS MANY TREATS TO EAT ON THIS DAY AS ON THE DAY BEFORE (pointing to the respective groups)?

If the child thinks they are different (focusing on the one counter or on the seven counters), ask whether they were the same before and what you did to make them different.

Make sure the child understands you're asking whether the total amount changed with a change in arrangement.

ACTIVITY FORM F: Equalizing Unequal Groups

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

MATERIALS: 22 counters of the same color.

PURPOSE: To see whether the child can focus on the subtraction of one group while at the same time adding to another, i.e., to examine addition and subtraction within parts of the whole.

PRESENTATION: Put out a pile of eight counters and another pile of 14 counters.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

WHICH PILE HAS MORE IN IT?

SEE IF YOU CAN MAKE IT SO  
THAT THERE ARE JUST AS  
MANY BEADS IN BOTH PILES.

If the child arranges the beads in a pattern to aid in counting, alter the child's arrangement and ask if they are still the same.

ACTIVITY FORM G: Dividing the Whole into Two Equal Groups

Name \_\_\_\_\_ Child's Name \_\_\_\_\_

Date \_\_\_\_\_ Child's Age \_\_\_\_\_

Child's Teacher \_\_\_\_\_

MATERIALS: Use 18 counters of one color.

PURPOSE: To examine how the child divides a whole into two equal parts.

PRESENTATION: Put out the pile of 18 counters.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

TAKE THESE BEADS AND MAKE A PILE FOR ME AND A PILE FOR YOURSELF. MAKE IT SO THAT WE BOTH HAVE EXACTLY THE SAME NUMBER OF BEADS.

After the child responds, probe to see if (and why) s/he regards them as equal. If the child arranges the beads in a pattern, try changing one of the patterns to see if this results in a change in judgment.

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## THE CONSERVATION OF LIQUID AND WEIGHT

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The child's understanding of conservation can also be assessed with liquid. In liquid conservation tasks, the child pours the same amount of water into two identical glasses. Then the interviewer pours the water of one of the glasses into a third glass of a different shape, and the child is asked whether the two glasses still have the same amount of water. The task can be carried out with a number of variations using a number of different sized containers.

It is generally a good idea to do the pouring for the child simply because children tend to spill small amounts, thus changing the quantities. Chapter 1 of The Child's Conception of Number provides a detailed description of liquid conservation and the responses of children in various stages of development.

One of the well documented facts in child development is that children come to understand the conservation of weight after they have acquired the conservation of substances such as liquid, clay, and so forth. Most children younger than nine or ten years of age do not conserve weight.

In the conservation of weight tasks, children are led to establish that two quantities, such as clay, are equal in weight. Then the shape of one of the two is changed and the child is asked whether its weight has changed or whether it is still the same. As always, it is important to ask children their reasons why the amounts are equal or different in weight.

ACTIVITY FORM H: Conservation of Liquid

Name \_\_\_\_\_ Child's Name \_\_\_\_\_

Date \_\_\_\_\_ Child's Age \_\_\_\_\_

Child's Teacher \_\_\_\_\_

**MATERIALS:** One quart each of two different colored liquids (use dye); two drinking glasses (A); four somewhat smaller glasses (B); four small glasses (C); one tall narrow glass (T).

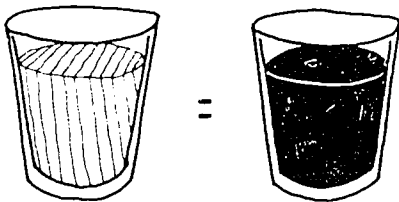
**PURPOSE:** To see how children establish equal amounts of water and whether they think an amount of liquid changes when poured into different shaped containers.

**PRESENTATION:** Put out both drinking glasses (A) and fill one 3/4 full.

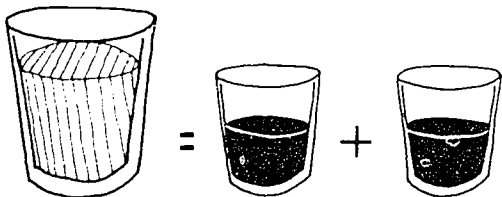
OUTLINE OF INTERVIEW

CHILD'S RESPONSES

PUT THE SAME AMOUNT OF WATER  
IN YOUR GLASS AS I HAVE IN MINE.  
(Help the child if necessary.)



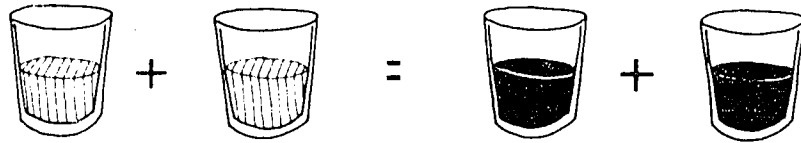
I'M GOING TO POUR MY WATER  
INTO THESE TWO OTHER GLASSES (B).  
Do the pouring. DO YOU STILL  
HAVE THE SAME AMOUNT AS I DO,  
MORE, OR LESS? WHY?



43

62

NOW I'M GOING TO POUR YOUR WATER INTO TWO GLASSES LIKE MINE.  
 Do the pouring. DO WE BOTH HAVE THE SAME AMOUNT OR DOES ONE  
 OF US HAVE MORE? WHY?



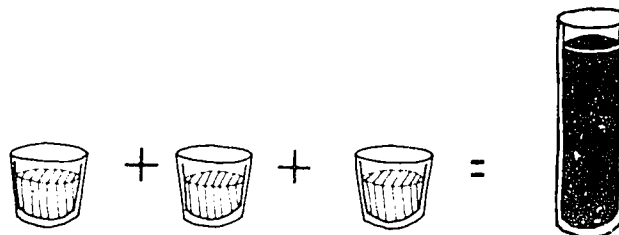
NOW I'M GOING TO POUR SOME OF MY WATER INTO THIS SMALL GLASS (C).  
 DO WE STILL HAVE THE SAME AMOUNT?



NOW I'M GOING TO POUR YOUR WATER INTO THESE THREE SMALL GLASSES (C).  
 DO WE HAVE THE SAME AMOUNT TO DRINK OR DOES ONE OF US HAVE MORE?



O.K., NOW I'M GOING TO POUR ALL OF MINE INTO THIS TALL GLASS (T).  
 HOW ABOUT NOW? DO WE HAVE THE SAME OF DIFFERENT AMOUNTS?



ACTIVITY FORM I: Conservation of Weight

Name \_\_\_\_\_ Child's Name \_\_\_\_\_  
Date \_\_\_\_\_ Child's Age \_\_\_\_\_  
Child's Teacher \_\_\_\_\_

MATERIALS: Same as in substance conservation (Activity 5, FORM B)--  
one cup each of playdough--plus a balance scale.

PURPOSE: To see if changes in appearances lead to judgments that  
weight has also changed.

PRESENTATION: Put out two balls of clay and the scale.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

HERE'S TWO BALLS OF CLAY. I WANT  
THE TWO BALLS TO WEIGH THE SAME  
AMOUNT. WHAT CAN YOU DO TO SEE  
IF THEY WEIGH THE SAME?

If they are not the same...

SEE WHAT YOU CAN DO TO MAKE  
THEM THE SAME. Have the  
child handle the balls and  
establish their equal weight.



When they are judged as weighing the same amount, take one of them and change its shape.

NOW WHAT DO YOU THINK? DO THEY BOTH WEIGH THE SAME OR DOES ONE WEIGH MORE THAN THE OTHER? WHY DO YOU THINK THEY'RE THE SAME (DIFFERENT)?

You can repeat the above by varying the alterations of one or both balls of clay.

If the child says they are the same, give a counter-probe.

ANOTHER CHILD TOLD ME THAT BECAUSE THIS ONE IS NOW THINNER (FATTER, ETC.) IT WEIGHS LESS (MORE). WHAT DO YOU THINK OF THAT?

OR

If the child says they are different weights, give a counter-probe.

ANOTHER CHILD TOLD ME THAT BECAUSE WE DIDN'T ADD OR TAKE ANY CLAY AWAY, THEY BOTH WEIGH THE SAME AMOUNT. WHAT DO YOU THINK OF THAT?

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## MEASUREMENT OF DISTANCE

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The following measurement task is the same as that shown in the videotape on quantitative relations. In this task, we see the child's use of quantitative notions in solving a clearly practical problem. Some interesting variations on this task can be found in Chapter 3 of Piaget in the Classroom.<sup>12</sup>

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12. Schwebel and Raph, Piaget in the Classroom.

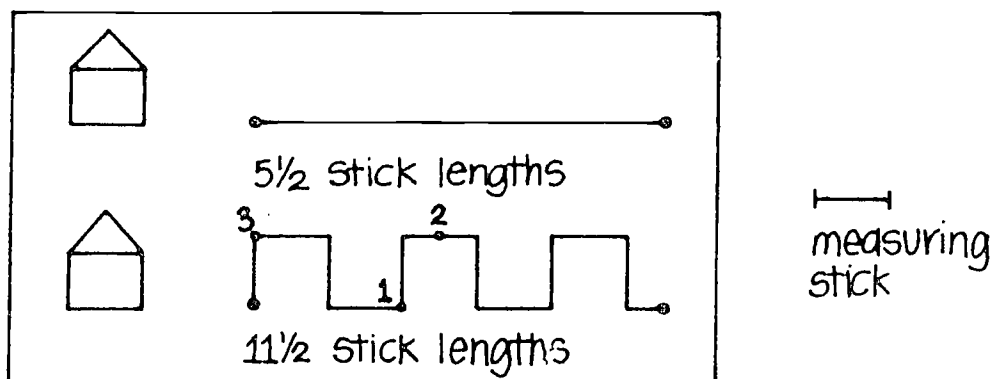
ACTIVITY FORM J: Measurement of Distance

Name \_\_\_\_\_ Child's Name \_\_\_\_\_

Date \_\_\_\_\_ Child's Age \_\_\_\_\_

Child's Teacher \_\_\_\_\_

**MATERIALS:** A one-foot by one-foot board (or paper) with two "roads" drawn on it; one straight and the other composed of short sections arranged at right angles; a measuring stick equal in length to one of the sections; two similar objects to move along the road.



**PURPOSE:** To see how the child measures distance--matching of beginning and ending points or use of a repeated measure.

OUTLINE OF INTERVIEW

CHILD'S RESPONSES

I'M GOING FOR A DRIVE ON THE CROOKED ROAD AND I GO ALL THE WAY TO HERE (Point 1). MOVE YOUR CAR SO YOU'VE GONE JUST AS FAR AS I HAVE. IS THAT JUST AS FAR? HAS YOUR CAR TRAVELED THE SAME DISTANCE AS MINE? Have the child explain. If the child doesn't get it right, ask if the measuring stick would help.

Then move your car to Point 2, and again ask the child to make his/her car go just as far-- travel the same distance. Point 2 should correspond in actual distance to the end of the straight road. Again, have the child explain his/her answer.

Last, go back to the beginning, moving both cars to the starting point. Move your car to Point 3, and ask the child to make his/her car go the same distance.

## **APPENDIX A:**

### **TRANSCRIPT OF VIDEOTAPE**

The Growing Mind. A Piagetian View of  
Young Children--The Development of  
Quantitative Relations: Conservation

As adults, we know that changing the arrangement of objects does not change their number. Eight objects remain eight objects irrespective of their arrangement. This principle is called conservation.

Research by Jean Piaget and others shows that most children under eight years of age do not conserve quantity. We will explore three stages in children's understanding of conservation between four and eight years of age.

Four year old Kevin is near the end of the pre-conceptual stage. Though there are nine yellow blocks and only eight blue ones, Kevin simply rearranges his to make the two groups appear equal in number.

Kevin's counting is accurate and he knows that the two groups are different. To correct the inequality he matches each of his blocks with a yellow one, setting up a one-to-one correspondence.

Even though each blue block is matched to a yellow one, Kevin counts to see if they are equal in number. While his use of one-to-one correspondence is somewhat advanced for his age and suggests that he is leaving the pre-conceptual stage, it does not convince Kevin that the two rows are equal in number.

During the pre-conceptual stage, children confuse quantity with space. Changing the spatial arrangement of objects changes their amount. Or in other words, what looks bigger is bigger.

Later in the interview, Kevin compares the two groups of blocks. He miscounts his nine blue ones.

Kevin attempts to reestablish the one-to-one correspondence between blue and yellow blocks. However, his understanding of one-to-one correspondence is not yet fully developed.

Bebe is seven years old and in the intuitive stage.

Bebe is not sure that the number of blocks is the same even though only their arrangement has changed. She counts to find out.

The yellow row is now longer but with more space between its blocks. We know that these changes cancel each other and number is conserved. Bebe cannot coordinate these relations and instead must count to see if the number has changed.

Shelly is almost eight and in the concrete operational stage.

She establishes a one-to-one correspondence between three sets of three blocks.

Without counting, Shelly knows that if nothing is added or subtracted the number remains the same.

Rather than counting, Shelly uses an argument of one-to-one correspondence. Her ability to reason about number without concern for spatial arrangement or need of counting characterizes the concrete operational stage.

Four year old Clare is questioned about the quantity of clay. This is a conservation of substance task. Children in the pre-conceptual stage believe that the amount of clay changes when its shape is altered.

Clare judged the two balls as equal. But when one was rolled out she thought there was less clay. She does not understand that while the snake was longer, it was also thinner and that these two physical changes cancelled each other, leaving the amount conserved.

Clare momentarily thought that the yellow ball was still less because it made a snake. However, when she rolled it, the yellow clay became equal in amount to the blue clay.

Earlier we saw that Bebe can count and determine whether two groups of blocks are equal in number. In this conservation of substance task, she knows that clay must be added or subtracted to make unequal amounts equal. This is an advance over pre-conceptual reasoning.

Earlier Bebe suggested adding clay to make unequal amounts equal. Yet she now thinks that changing the shape changes the amount.

The child's conception of conservation develops with an anticipation that changes in one dimension can be compensated by changes in another. Bebe does not understand that as the blue clay becomes larger in circumference, it also becomes thinner. She focuses only on the large circumference and therefore judges the blue clay as more.

Eight year old Robin knows that the amounts are still equal since they were equal before. This implies an understanding that while the ball became thinner and longer to form a snake, the snake can become fatter and shorter to form the original ball. An understanding that changes in one dimension can be counteracted by those in another characterizes the concrete operational stage.

Four year old Shana is in the pre-conceptual stage.

The blue one extends further out on one side and is therefore judged longer.



At the pre-conceptual stage, children have no way of understanding that spatial changes can compensate each other. While the straight blue wire extended beyond the end of the red one, this was compensated by the red extending beyond the other end of the blue wire.

The bent wire is judged as little because it matches the red wire at one end but is shorter at the other. This judgment does not take account of the relationship between the bends in the wire and its appearance of shortness.

Because both ends match, Shana knows that they are the same length.

Seven year old Bebe does not conserve substance or number even though she can count. To further investigate her quantitative reasoning she is given a length measurement problem.

Unlike Clare, Bebe has an intuitive sense that the crooked path is longer. However, this understanding is not yet accompanied by a conception of units. Rather than thinking of the crooked path as an addition of shorter distances, she treats the two paths as if they are straight and places her doll so that the two are exactly opposite each other. The stick is used to emphasize this placement.

Bebe's conviction is firm. She cannot be convinced that equal distances imply an equal number of units. For her, equal distances require equivalent starting and ending points.

Earlier we saw that eight year old Robin conserved substance. This indicates some understanding of units.

Using the stick as a unit of measure, Robin knows that the same number of added units implies the same distance.

Robin not only knows that one path is longer, but she can also accurately predict an equal distance.

Robin shows that she can use fractions of units and that she can subtract units as well as add them.

Robin knows that one segment of the crooked path is equal to the stick and one corresponding segment on the straight path is also equal to the stick.

During the pre-conceptual stage, the concept of quantity is tied to how things appear. When objects are rearranged, children in this stage think their number has also changed. Making a row shorter is thought to reduce the number of blocks. To think otherwise requires understanding that while one row is shorter, there is also less space between its blocks.

This same type of reasoning is seen in the conservation of substance task. When one ball is changed into a snake, Clare thinks the amount has also changed. She does not understand that while one ball lengthened, it also became thinner.

Children in this stage also fail to conserve length. If one wire extends beyond the end of another, it is thought to be longer. The pre-conceptual child does not take account of the fact that it is longer at one end but shorter at the other.

Between six and seven years of age, most children enter the intuitive stage. Conservation is now considered a possible but not logically necessary condition. Bebe counts to find out whether the spatial rearrangement of the blocks has also changed their number.

Most children in the intuitive stage can count. But this does not necessarily mean that they have acquired the concept of conservation. Bebe can count blocks to determine whether their number has changed, but she does not conserve substance.

A child's inability to conserve quantity implies that they can not yet use units of measure in their thinking. Bebe does not think of distance as a sum of shorter distances or units. Instead, she places her doll so that the two are physically close to each other.

By eight, most children enter the concrete operational stage and realize that one type of change can compensate the effects of another. This insight provides the psychological basis for an understanding of conservation. Shelly knows that additions and subtractions can compensate each other. This, in turn, leads to an understanding that if nothing is added or subtracted, equal amounts remain equal.

The child's understanding that one operation can reverse the effects of another is called reversibility. Reversibility of thought is expressed in a number of ways. Robin knows that changes in thickness are compensated by changes in length. She also knows that the snake must be equal to the blue clay since it could be changed back into an equal sized ball.

Conservation provides a basis for understanding units of measure. Robin uses the stick to measure the number of units in the crooked path. She knows that her doll must travel the same number of stick lengths of the straight one.

During early childhood, the child's intelligence goes through a series of complex developmental changes. This occurs gradually and is shown in this film by the changing views of conservation and measurement.

Research by Jean Piaget and others indicates that this development is not the direct result of instruction or maturation. Each level of understanding is constructed by the child in the course of active interactions in the social and physical world. Only by the child's application of his or her own thought will earlier forms of thinking be modified into more complete and systematic ways of understanding the world.

## **APPENDIX B:**

### **REFERENCES AND ADDITIONAL RESOURCES**

## Some of Piaget's Translated Works

These earliest of Piaget's books concern the general character of children's thinking between three and eight years of age, as revealed in either natural observations or discussions with children. At the time of their writing, Piaget did not regard them as important works, and he later criticized them for their dependence on the child's verbal reasoning. However, these books set the stage for much of his later work and provided the foundation for public interest. Of his major descriptive works, these are probably the most readable. The works are listed in order of their original French publication dates.

- (1923) The Language and Thought of the Child. New York: Meridian, 1955.
- (1924) Judgment and Reasoning in the Child. New Jersey: Littlefield, Adams and Co., 1966.
- (1926) The Child's Conception of the World. New Jersey: Littlefield, Adams and Co., 1965.
- (1927) The Child's Conception of Physical Causality. New Jersey: Littlefield, Adams and Co., 1965.
- (1932) The Moral Judgment of the Child. New York: Collier, 1962.

These three books express Piaget's major observations on the mental development of infants. The Origins of Intelligence provides a theoretical model of sensory-motor intelligence. The Construction of Reality describes the first understandings of space, time, objects, and causality. The original French title, Origins of the Symbol, suggests the underlying focus of Play, Dreams, and Imitation in Childhood. Each of these three books will probably disappoint the casually interested reader. The Construction of Reality is the easiest of the three to read.

- (1936) The Origins of Intelligence in Children. New York: Norton, 1963.
- (1937) The Construction of Reality in the Child. New York: Basic Books, Inc., 1954.
- (1946) Play, Dreams, and Imitation in Childhood. New York: Norton, 1962.

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- These works provide activities and/or information on the development of quantitative relations.

The following books make up the largest single focus in the study of cognitive development. In them, Piaget explores the development of logical and sub-logical thought between four and 12 years of age and its expression in a broad cross-section of knowledge. Each is composed of a rich array of concrete-manipulative experiments and the corresponding responses of children. The Growth of Logical Thinking is Piaget's major work on formal-operational thought. The beginning Piaget student will find the descriptions rich and readable, though tedious. The theoretical accounts are highly abstract and complex.

- (1941) The Child's Conception of Number. New York: Norton, 1965.
- (1946) The Child's Conception of Movement and Speed. New York: Ballantine, 1971.
- (1946) The Child's Conception of Time. New York: Basic Books, Inc., 1969.
- (1948) Piaget, Inhelder, and Szeminska, The Child's Conception of Geometry. London: Routledge and Kegan, 1960.
- (1948) Piaget and Inhelder, The Child's Conception of Space. New York: Norton, 1967.
- (1951) -----, The Origin of the Idea of Chance in Children. New York: Norton, 1975.
- (1955) Inhelder and Piaget, The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books, Inc., 1958.
- (1959) -----, The Early Growth of Logic in the Child. New York: W. W. Norton and Co., Inc., 1964.

The following books provide an overview of Piaget's theory and his general views on the nature of knowledge. The Psychology of the Child provides his best introductory overview of development between infancy and late adolescence. As suggested by their titles, two of the books present Piaget's thoughts on education. They do not provide simple educational prescriptions.

- (1939 & 1965) Science of Education and the Psychology of the Child. New York: Viking, 1971.
- (1947) The Psychology of Intelligence. New Jersey: Littlefield, Adams and Company, 1963.
- (1948) To Understand is to Invent: The Future of Education. New York: Grossman, 1973.
- (1964) Six Psychological Studies. New York: Vintage Books, 1967.

- (1966) Piaget and Inhelder, The Psychology of the Child. New York: Basic Books, Inc., 1969.
- (1968) Structuralism. New York: Harper and Row, 1971.
- (1973) The Child and Reality: Problems of Genetic Psychology. New York: Grossman, 1973.

### **Piaget as Seen by Others**

These five books provide an overview of Piaget's theory and his main findings. Pulaski's and Phillips' works are probably the most readable by lay persons. The book by Ginsburg and Opper is an excellent overview of the main stages of development from infancy to late adolescence. The books by Boyle and Flavell focus more on the formal aspects of Piaget's theory and are probably more useful to the advanced student. Flavell's book is a classic American interpretation of Piaget's general theory.

- Boyle, D. G., A Student's Guide to Piaget. London/New York: Pergamon Press, 1969.
- Flavell, John H., The Developmental Psychology of Jean Piaget. Princeton, New Jersey: Van Nostrand, with a foreword by J. Piaget, 1963.
- Ginsburg, Herbert, and Opper, Sylvia, Piaget's Theory of Intellectual Development: An Introduction. New Jersey: Prentice-Hall, 1969.
- Phillips, John L., Jr., The Origins of Intellect. San Francisco: W. H. Freeman, 1969.
- Pulaski, Mary Ann Spencer, Understanding Piaget: An Introduction to Children's Cognitive Development, New York: Harper and Row Inc., 1971.

The book by Isaacs is a good introductory presentation of quantitative concepts (number, measurement, time, etc.) between four and eight years of age. Brearley and Hitchfield provide a similar treatment of additional topics such as space, morality and science.

- Brearley, Molly and Hitchfield, Elizabeth, A Guide to Reading Piaget. New York: Schocken Books, 1966.
- Isaacs, Nathan, A Brief Introduction to Piaget. New York: Agathon Press, 1972.



Dasen's article explores the relationship between culture and knowledge. Furth provides a rich and insightful presentation of Piaget's general theory. He includes seven short papers by Piaget. Langer describes three predominate views on mental development: behaviorist, structuralist, and analytic. Ripple and Rockcastle edited the presentations of a large American conference on Piaget. They include four papers by Piaget, a number of theoretical papers on education, and a large number of papers concerning curriculum projects based on Piaget's theory. The papers by Piaget are informative and quite readable.

Dasen, Pierre R., Biology or Culture? Interethnic Psychology from a Piagetian Point of View, Canadian Psychologist, April 1973, 14 (2), 149-166.

Furth, Hans G., Piaget and Knowledge: Theoretical Foundations. New Jersey: Prentice-Hall, 1969.

Langer Jonas, Theories of Development, San Francisco: Holt, Rinehart and Winston, Inc., 1969.

- Ripple, Richard R. and Rockcastle, Verne N. (Eds.), Piaget Rediscovered: A Report of the Conference on Cognitive Studies and Curriculum Development. Ithaca: School of Education, Cornell University, 1964.

These works reflect some of the research studies directed toward refining and clarifying Piaget's theory and its implications.

- Almy, Millie, with Chittenden, E. and Miller, P., Young Children's Thinking: Studies of Some Aspects of Piaget's Theory. New York: Teachers College Press, Columbia University, with a foreward by J. Piaget, 1966.
- -----, and Associates, Logical Thinking in Second Grade. New York: Teachers College Press, Columbia University, 1970.
- Dasen, Pierre R., Cross-Cultural Piagetian Research: A Summary, Journal of Cross-Cultural Psychology, 7, 1972, 75-85.
- Elkind, David and Flavell, John H. (Eds), Studies in Cognitive Development: Essays in Honor of Jean Piaget. New York: Oxford University Press, 1969.
- Hyde, D. M. G., Piaget and Conceptual Development: With a Cross-Cultural Study of Number and Quantity. London: Holt, Rinehart, and Winston, 1970.
- Kofsky, Ellin, A Scalogram Study of Classificatory Development. Logical Thinking in Children: Research Based on Piaget's Theory. Irving Siegel and Frank Hooper (Eds.), New York: Holt, Rinehart, and Winston, Inc., 1968.

- Siegel, Irving, and Hooper, Frank (Eds.), Logical Thinking in Children: Research Based on Piaget's Theory. New York: Holt, Rinehart, and Winston, Inc., 1968.

## **Piaget's Theory and Education**

The following books and papers present a range of views on the general implications of Piaget's theory for education. The book by Schweibel and Rath presents a number of readable and excellent articles by various Piagetian scholars.

Alward, Keith R., The Implications of Piaget's Theory for Day-Care Education. Child Care: A Comprehensive Guide. Stevanne Auerbach Fink (Ed.), New York: Behavioral Publications, 1973.

-----, A Piagetian View of Skills and Intellectual Development in the Responsive Model Classroom. Non-published paper, Far West Laboratory, 1973.

Duckworth, Eleanor, Piaget Takes a Teacher's Look, Learning, October 1973.

Furth, Hans G., Piaget for Teachers. New Jersey: Prentice-Hall, 1970.

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Weikart and the Responsive Program Staff present two different broad applications of Piaget to early childhood education. Both are models for the National Follow Through Program. The Responsive Model Program has been implemented in hundreds of classrooms throughout the U.S. The paper by Rayder, et al., presents some of the findings on the effects of the program.

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## **Films and Videotapes**

CRM Educational Films, Cognitive Development. (20 minutes)  
Available from CRM Educational Films, 7838 San Fernando Road, Sun Valley, CA 91352.

Davidson Films, Piaget's Developmental Theory:

- Classification. (19 minutes)
- Conservation. (29 minutes)
- Formal Thought. (33 minutes)

- Growth of Intelligence in the Pre-school Years. (31 minutes)  
Jean Piaget: Memory and Intelligence. (44 minutes)  
 Available through the University of California Extension  
 Media Center, Berkeley, CA 94720.
- Far West Laboratory, The Growing Mind: A Piagetian View of Young  
 Children:
  - The Development of Classification. (30 minutes)
  - The Development of Order Relations -- Seriation. (27 minutes)
- The Development of Quantitative Relations -- Conservation.  
 (32 minutes)  
The Development of Spatial Relations. (29 minutes)  
 Available through the Far West Laboratory, 1855 Folsom  
 Street, San Francisco, CA 94103.
- Phoenix Films, Learning About Thinking and Vice Versa. (32 minutes)  
 Available through Phoenix Films, 743 Alexander Road,  
 Princeton, NJ 08501.
- The Jean Piaget Society, Equilibration. (35 minutes) Available  
 through The Jean Piaget Society, Box 493, Temple University,  
 Philadelphia, PA 19122.



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42

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